JOURNAL OF APPLIED POLYMER SCIENCE VOL. 7, PP. 153-169 (1963)

Effect of Radiation on Tension, Compression, Bending, and Shear Properties of Polyethylene

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STRESS-STRAIN PROPERTIES

INTRODUCTION

The main purpose of this investigation was to determine the influence of nuclear radiation on the stress-strain properties of polyethylene for tension, compression, bending, and shear stresses. The material tested was a low-density polyethylene, Polyethylene 75, an experimental polymer produced by E. I. Du Pont de Nemours & Company. Low-density polyethylene is a long-chain linear polymer consisting of only carbon and hydrogen atoms. It was selected for this investigation because it has a short radioactivity period after irradiation, contamination and disposal problems being thus eliminated.

To investigate the influence of radiation, five sets of tests for each type of stress were conducted. Four sets of tests were made with irradiation doses of 1.0×10^{17} nvt, 2.5×10^{17} nvt, 5.0×10^{17} nvt and 7.5×10^{17} nvt. For reference purposes, a fifth set of tests was conducted with no irradiation applied. The symbol nvt is the dosage unit, where nv refers to n neutrons per cubic centimeter moving with a velocity of v centimeters per second in a given direction. Then the product nv (called the neutron flux) is the number of neutrons of velocity v which cross one square centimeter of a material per second. The symbol nvt represents the dosage unit and represents the total number of neutrons which have crossed a unit surface of the material.

Specimens

For all types of tests, three check tests were conducted for each radiation dose. The specimens were prepared by molding according to ASTM specifications.[1] The dimensions of the four kinds of specimens used in this study are given in Figure 1. The specimens were irradiated in a swimming-pool type of reactor (available as a University reactor facility).

Test Procedures

Tests were conducted in a laboratory with a controlled temperature of 73 ± 2°F. and a humidity of 50 ± 2% R. H. The short-time tension

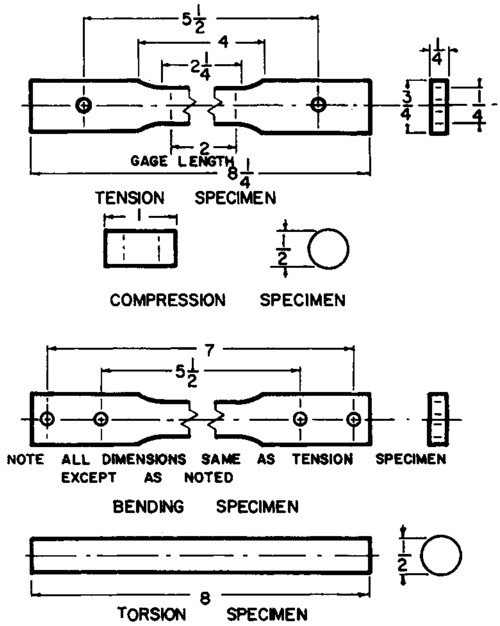

Fig. 1. Test specimens.

tests were made with an Instron Tester having a head speed of 0.2 in./min. The strains were measured with a pair of dividers and a scale with divisions of one-hundredth of an inch. The compression, bending, and torsion tests were made with a special creep machine, the loads being applied manually. Mechanical gauges were used to measure the deformations for each type of stress investigated. In the case of torsion, the angle of twist was measured for a selected gauge length at each load increment, while for the pure bending tests, deflections for a selected gauge length were recorded at each load increment.

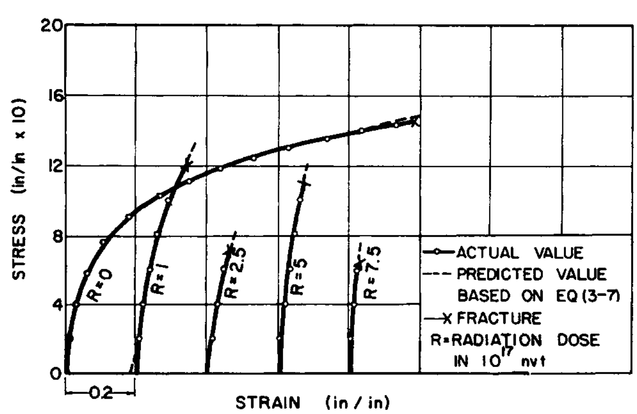

Fig. 2. Nominal tension stress–strain for different radiation doses for Polyethylene

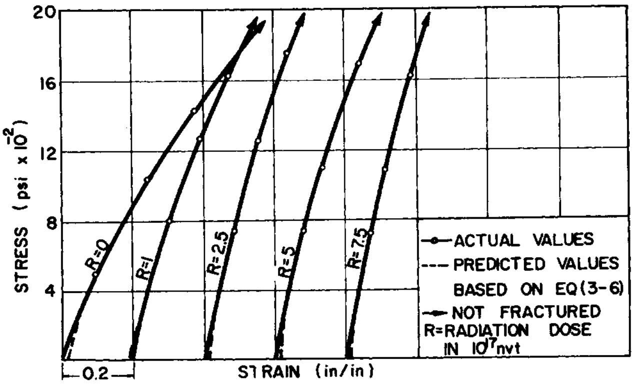

Fig. 3. Nominal compression stress–strain for different doses of radiation for Polyethylene 75.

Test Results

The average nominal stress–strain curves in tension and compression are shown in Figures 2 and 3 for each of the five radiation doses. For the pure bending tests, curvature values (equal to the reciprocal of the radius of curvature) were determined from the deflection values and gauge length. Plots of the moments versus the curvature for each of the radiation doses are given by the solid lines in Figure 4. Figure 5 represents the average torque moment versus angle of twist relations for the five radiation doses. All curves shown in Figures 2, 3, 4, and 5 represent the averages of three specimens. Based upon the results given in Figures 2 to 5, the various stress–strain properties can be determined as described in the following.

Nominal Tension Stress–Strain Properties

Based on Figure 2, the nominal stress–strain properties can be determined.[2a] The nominal *fracture* and *ultimate strength* values correspond to

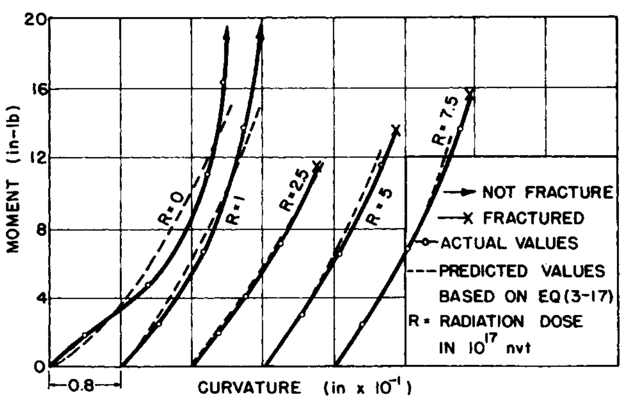

Fig. 4. Short-time bending moment–curvature for different radiation doses for Polyethylene 75.

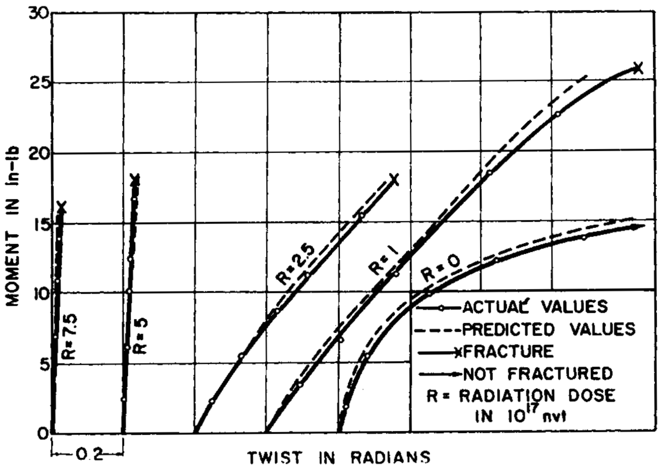

Fig. 5. Torque–twist diagrams for different doses of radiation for Polyethylene 75.

the maximum stress values in Figure 2. An *elastic* strength was determined, based upon a modified Johnson's apparent elastic limit corresponding to a point where the rate of change of strain is 50% greater than the initial tangent modulus. The *deformation* properties determined were the *ductility*, or per cent strain at fracture, and the *stiffness*, defined as the initial tangent modulus of elasticity corresponding to the slope of the stress–strain curve at the point of zero stress. The *energy* properties determined in-

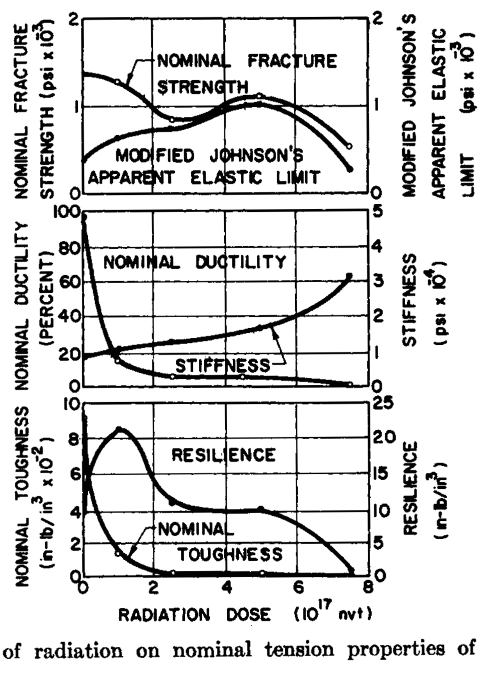

Fig. 6. Effects of radiation on nominal tension properties of Polyethylene 75.

clude the toughness (equal to the strain energy per unit volume needed to stress the material to fracture) and the resilience (equal to the strain energy per unit volume versus the modified Johnson's elastic limit, defined above). The foregoing strain energies are equal to the areas under the stress–strain up to the fracture and the modified Johnson's elastic limit stresses, respectively.

With the data in Figure 2 and the foregoing definitions of the mechanical properties, the six properties represented in Figure 6 were determined for the five radiation doses. An examination of this figure shows that the optimal radiation dose differs in accordance with the property considered.

True Tension Stress–Strain Properties

The true stress and true strain based on the actual cross-sectional area and changing gauge length have been defined respectively by eq. (2):

$$S = P/A \tag{1}$$

$$\delta = \ln (1 + \epsilon) \tag{2}$$

where A is the actual area at the load P and ϵ is the nominal strain.

Assuming constancy of volume,

$$A_0L_0 = AL \tag{3}$$

where A_0 and L_0 are the initial cross-sectional area and gauge length respectively, and A and L are the cross-sectional area and gauge length respectively, at the load P.

From eqs. (1) and (3):

$$S = P/A = PL/A_0L_0 = (P/A_0)(1 + \epsilon) \tag{4}$$

The true tension stress–strain curves based upon the true stress and strain values defined by eqs. (2) and (4) are plotted in Figure 7 for the various radiation doses. From Figure 7 the true fracture strengths, true ductility,

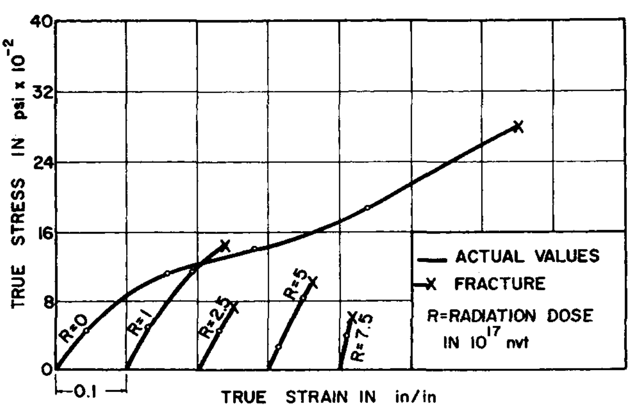

Fig. 7. Tension true stress vs. true strain for different radiation doses for Polyethylene 75.

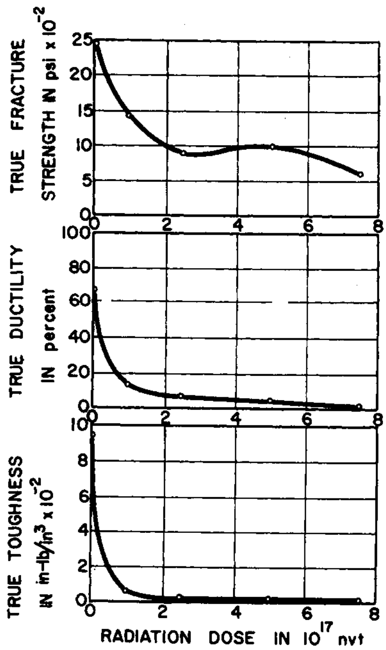

Fig. 8. Effect of radiation on true tension properties of Polyethylene 75.

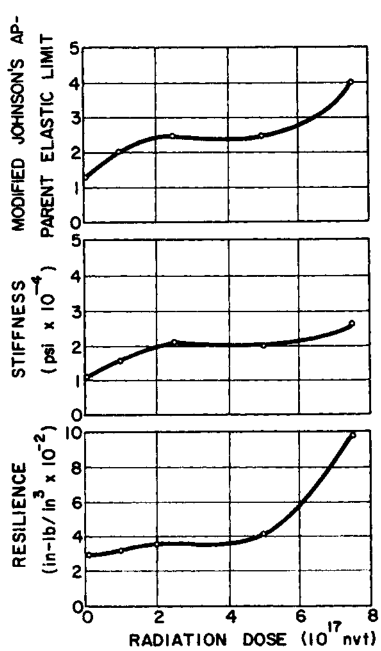

Fig. 9. Effects of radiation on nominal compression properties of Polyethylene 75.

and true toughness values can be determined. Variation of these true property values with radiation dose are given in Figure 8.

Nominal Compression Stress–Strain Properties

Nominal compressive stress–strain curves representing the average of three specimens are shown in Figure 3 for the five radiation doses considered. From these curves, only the elastic properties of elastic strength, stiffness, and resilience could be determined, since the specimens could not be fractured. Variation of these three properties with radiation dosage are shown in Figure 9.

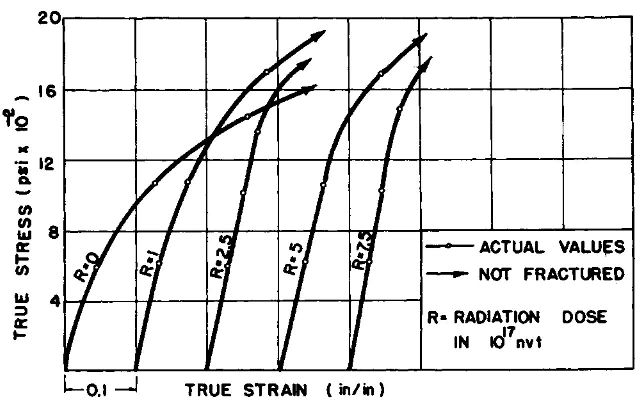

Fig. 10. Compression true stress vs. true strain for different doses of radiation for Polyethylene 75.

True Compression Stress–Strain Properties

With eqs. (2) and (4) the true compression stress–strain curves can be determined, as shown in Figure 10. Because in the elastic range the nominal and true stresses and strains are essentially equal, there is little difference between the nominal and true properties. Since the specimen could not be fractured in compression, true plastic properties could not be evaluated.

Nominal Stress–Strain Relations in Tension and Compression

One of the objectives of this study was to predict the bending and shear stress–strain relation from the tension and compression results. For this purpose, an empirical relation was found (for both tension and compression) which gives a good approximation for the nominal stress–strain relation. As shown in Figure 11, it was found that the logarithm of the strain versus the stress gave linear relations. The equation for the stress–strain relations can then be expressed as

$$\epsilon = ke^{BS} + \epsilon_0 \tag{5}$$

where e is the base of natural logarithms and k, B, and ϵ_0 are material constants.

Figures 2 and 3 show that, except for small stress values, there is good agreement between the actual stress–strain curves and the predicted curves based on Eq. (5).

Nominal Bending Properties

To determine the effects of radiation on the bending properties of polyethylene, the mechanical properties in bending were determined from the bending moment–curvature curves for each radiation dose (Fig. 4). The curvature is defined as the reciprocal of the radius of curvature. The

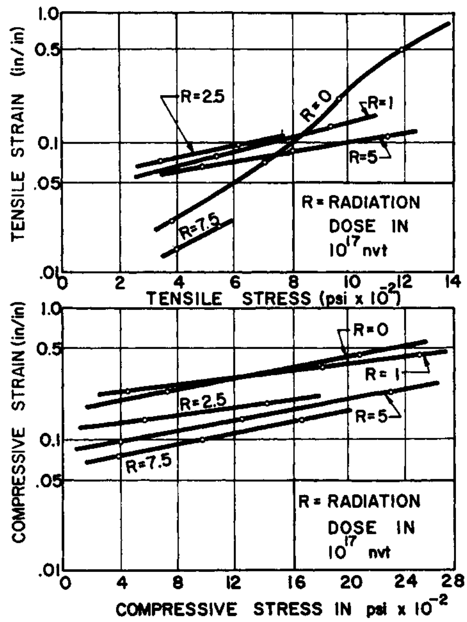

Fig. 11. Nominal tension and compressive stress–strain plotted on semilog basis.

properties obtained were plotted against the radiation dose. The specimens could be fractured only for the three highest radiation doses, so that a maximum load and modulus of rupture could be evaluated only for three radiation doses. The values of the modulus of rupture were found with the equation $S = Mc/I$, where M is the maximum fracture moment, I is the moment of inertia, and c is the distance to the outer fiber. Ductility values in bending were found by measuring the maximum deflection values corresponding to the maximum load. Toughness values were found by the average work done per unit volume: T_b = area under the moment versus angle of rotation, divided by the volume of the specimen.

The *stiffness* in bending is generally defined by the modulus of elasticity E_t as determined by

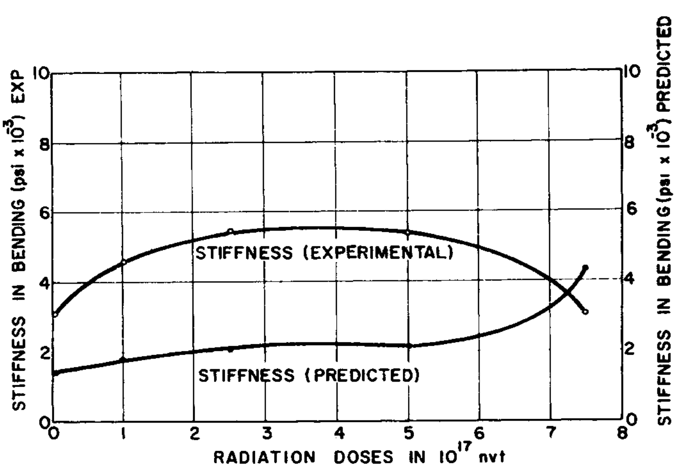

Fig. 12. Radiation effect on experimental and predicted stiffness in bending of Polyethylene 75.

$$E_b = ML^3/SyI \tag{6}$$

where y is the deflection for the length L and M is the pure bending moment applied to the ends of the specimen.

For materials with different moduli in tension and compression, it can be shown[3] that the modulus in bending is

$$E_b = 4\, E_t E_c/(\sqrt{E_t} + \sqrt{\mathrm{E}_l})^2 \tag{7}$$

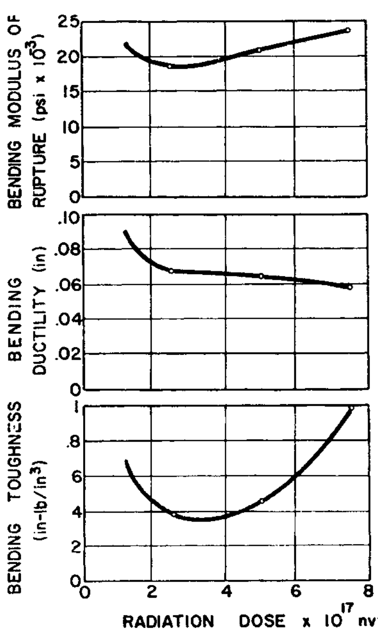

Fig. 13. Effects of radiation on bending properties of Polyethylene 75.

where E_t and E_l are respectively the moduli of elasticity in tension ar-d -om-pression. A comparison of the modulus values E_b, as defined by eqs. (6) and (7), is shown in Figure 12. Figure 12 also shows the variation of bending stiffness with radiation dose, while Figure 13 shows the variation of plastic bending properties of modulus of rupture, ductility, and toughness with radiation dose. The nature of the moment–
Figure 4 prevented the evaluation of elastic strength a

An examir ion of Figures 12 and 13 shows that t
and toughness increased, while the ductility decre
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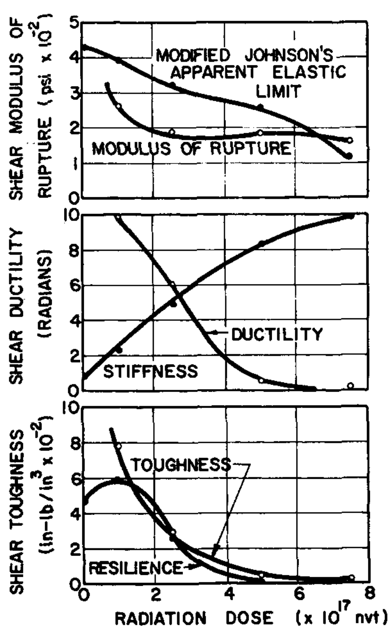


Fig. 14. Effects of radiation on shear properties of Polyethylene 75.

Nominal Torsion Properties

To determine the effect of radiation dose on the torsion properties, torque versus angle of twist was determined for each radiation dose, as shown in Figure 5. From these torque–twist diagrams both the elastic and plastic shear properties were determined, as described in the following.

The elastic shear strength was determined approximately from $S_{sy} = T_y r/J$, where the yield torque T_y is the torque at a slope in the T–θ curve which is 50% greater than at the origin. The *stiffness* in shear is measured approximately by $E_s = T_y L/(\theta_y J)$ where θ_y is the angle of twist corresponding to T_y. The resilience is calculated from the average modulus of elastic resilience $U_s = (T_y\theta)/(2AL)$ which is the total strain energy corresponding to T_y divided by the volume AL.

The plastic strength is measured by the modulus of rupture $S_{su} = T_u rJ$, where T_u is ultimate, or fracture, torque. The ductility in shear was determined by the angle of twist at fracture, while the toughness values were evaluated as the average work per unit volume needed to fracture the specimens, or:

$$\int_0^{\theta_f} Td\theta/(AL).$$

The variations of the foregoing six properties with radiation dose are shown in Figure 14. An examination of this figure shows that the values of all properties except stiffness decreased with increase in radiation dose.

PREDICTION OF MOMENT–CURVATURE RELATIONS FROM TRUE TENSION AND COMPRESSION STRESS–STRAIN CURVES

Moment–curvature relations in flexure can be obtained from tension and compression stress–strain relations by methods described by Nadai.[2b] In applying Nadai's graphical procedure, it was found necessary to use true stress and true strain values, since nominal stress and strain values were found to provide inadequate accuracy. To develop the necessary equations for predicting the moment–curvature relations, consider a prismatic bar of width b and depth h subjected to a pure moment M at each end. If the tension and compression properties of the material are different, then the neutral axis is not at mid-depth but is located at a distance y_1 from the bottom fiber and y_2 from the top fiber. Then, if the bending stress at a distance y from the neutral axis is σ, for equilibrium:

$$\int_{y_1}^{y_2} \sigma dA = 0 \tag{8}$$

and

$$\int_{y_1}^{y_2} \sigma y dA = M \tag{9}$$

where σ = nominal stress, dA = differential element of area, and M = the bending moment.

If ϵ is the nominal strain at the distance y from the neutral axis, and R is the radius of curvature, for plane sections to remain plane:

$$\epsilon = y/R \tag{10}$$

For a rectangular cross section, $dA = bdy$ and eq. (9) becomes:

$$\int \sigma \epsilon R b dy. = M \tag{11}$$

Placing the value of $dy = Rd$, from eq. (10), into eq. (11):

$$bR^2 \int_{\epsilon_u}^{\epsilon_t} \sigma \epsilon d\epsilon = M \tag{12}$$

From eqs. (2) and (4), the values of the nominal stress and strain in terms of the true stress and strain are

$$\epsilon = e^{\delta} - 1 \tag{13}$$

$$\sigma = s/1 + \epsilon = Se^{-\delta} \tag{14}$$

Placing values of nominal stress and nominal strain from eqs. (13) and (14) into eq. (12):

$$bR^2 \int_{\delta_u}^{\delta_t} S(e^\delta - 1)\, d\delta = M \qquad (15)$$

or

$$bR^2\left[\int_0^{\delta_t} S\epsilon d\delta - \int_0^{\delta_c} S\epsilon d\delta\right] = M \qquad (16)$$

In eq. (16) the integrals are simply curves of the areas under the $S\epsilon - \delta$ for simple tension and compression.

For a rectangular cross section, from the first condition of equilibrium eq. (8) and eq. (10):

$$\int \sigma dA = \int \sigma b dy = \int \sigma b R d\epsilon = 0$$

or

$$\int_{\epsilon_c}^{\epsilon_t} \sigma d\epsilon = 0 \qquad (17)$$

Placing values of σ and $d\epsilon$ from eqs. (13) and (14) into eq. (17):

$$\int_0^{\delta_t} S d\delta = \int_0^{\delta_c} S d\delta \qquad (18)$$

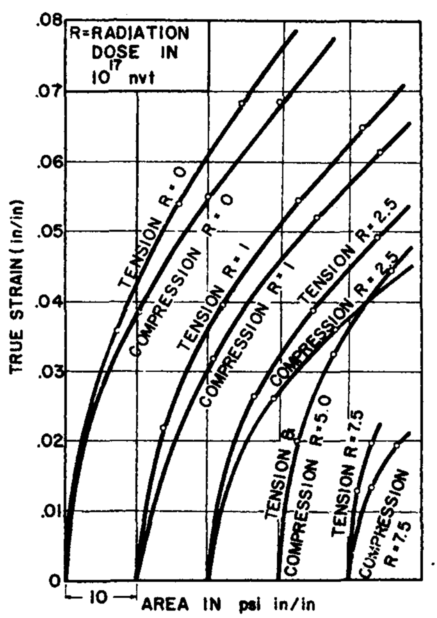


Fig. 15. Areas under true stress vs. true strain for different radiation doses for Polyethylene 75.

Equation (18) simply states that to satisfy the equilibrium condition $\Sigma F = 0$, the areas under the true stress–strain curves in simple tension and compression must be equal. Now, using the true stress–strain diagrams for selected values of true strain δ, values of areas $\int S d\delta$ and $\int S\epsilon d\delta$ can be determined. These areas are plotted as functions of strain in Figures 15 and 16. For a given value of δ_t, eq. (18) and Figure 15 provide a means of finding the corresponding value of δ_c. Then from Figure 16, and for these values of δl and δl, the areas $\int S\epsilon d\delta$ can be found. Placing these values of area in eq. (16) then yields the value of curvature R for the selected δ_t and M values. Repeating the foregoing procedure for a series of values of δ_t then gives a number of values of R corresponding to M, from which $M - R$

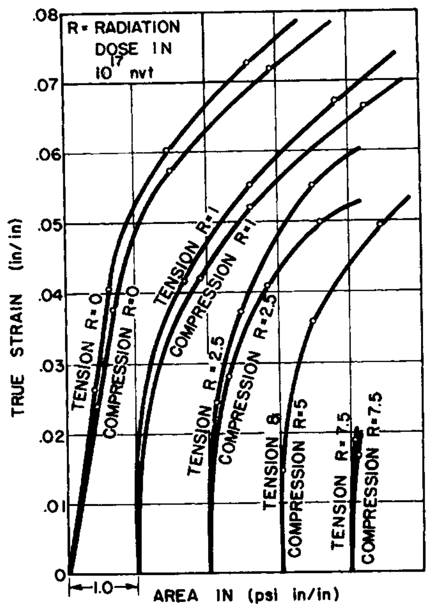


Fig. 16. Areas under true stress vs. true strain for different radiation doses for Polyethylene 75.

theoretical curves can be plotted, as shown in Figure 4. A comparison of the theoretically determined values of curvature R with their experimental values, shown in Figure 4, indicates that the theoretical values based upon the true tension and compression stress–strain curves provides a good prediction of the curvature values.

PREDICTION OF TORQUE-TWIST RELATIONS FROM TRUE TENSION AND COMPRESSION STRESS–STRAIN CURVES

Torque–twist relations can be predicted from tension and compression stress–strain curves by means of a modification of Nadai's method[2b] and a procedure derived by Marin and Pao[4] for defining shear strains for a material with different properties in tension and compression. To find the torque (t) versus twist (θ) relation, consider the circular cylindrical specimen of length L and radius R and let the shear strain at an intermediate radius r be γ. Then:

$$\gamma = r\theta \tag{19}$$

where $\theta = \theta'/L$; θ' is the angle of twist for a length L and θ is the angle of twist per unit length.

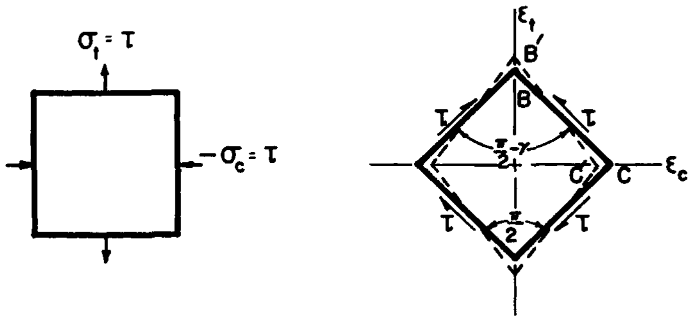
If τ is the shear stress at a radius r, then for equilibrium:

$$T = \int_0^R (2\pi r dr)(\tau r) = 2\pi \int_0^R \tau r^2 dr \tag{20}$$

Noting that $r = \gamma/\theta$ and $dr = d\gamma/\theta$, from eq. (20):

$$T = (2\pi/\theta^3) \int_0^\gamma \tau\gamma^2/d\gamma \tag{21}$$

Eq. (21) states that the torque T is equal to the moment of inertia of the area under the shear stress–strain about the stress axis (the integral in eq. 21) multiplied by $2\pi/\theta^3$. In computing the integral in eq. (21), the shear stress–strain diagram must first be found from the tension and compression relations. To do this, the case of torsional or pure shear stress, shown in Figure 17*b*, is replaced by the equivalent biaxial principal stress system, as shown in Figure 17*a*. If the shear strain is γ and the principal



(a) (b)

Fig. 17. Deformation of an element in shear.

strains corresponding to the principal stresses σ_t and σ_c are ϵ_t and ϵ_c, then, as is shown by Marin and Pao,[4] for materials with different properties in tension and compression:

$$\gamma = [2(\epsilon_t + \epsilon_c)]/(2 + \delta_t - \epsilon_c) \tag{22}$$

For $\epsilon_c = \epsilon_t = \epsilon$, from eq. (22), $\gamma = 2\epsilon$, which is the value generally used for the shear strain. For large strains, the nominal strains ϵ_c and ϵ_t should be replaced by the true strains δ_c and δ_t, and eq. (22) becomes:

$$\gamma = (2(\delta_t + \delta_c))/(2 + \delta_t - \delta_c) \tag{23}$$

The shear stress–strain diagram can now be obtained by using the tension and compression true stress–strain diagrams. For selected values of $\tau = \sigma_c = \sigma_t$, and from the stress–strain curves, corresponding values of δ_c and δ_t can be calculated. Then by eq. (23) the value of the shear strain γ for the selected value of the shear stress τ is calculated. Repeating the foregoing procedure for a series of points provides the data for plotting the shear stress–strain curves shown in Figure 18. Using this figure, the mo-

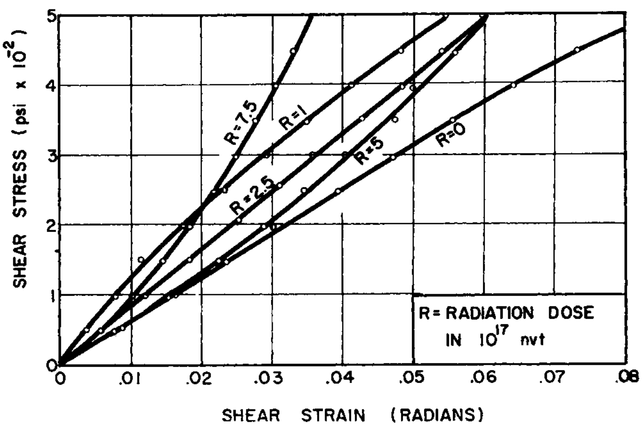


Fig. 18. Shear stress vs. shear strain based on true stress vs. true strain relations.

ment of inertia values $\int \gamma^2 \tau d\gamma$ can be found for selected values of the shear strain γ. A planimeter was used to determine the moment of inertia of the areas under the shear stress–strain. Plots of γ versus $\int \gamma^2 \tau d\gamma$ are given in Figure 19. Using moment of inertia values $\int \gamma^2 \tau d\gamma$, as given in Figure 19, the values of torque T for each value of θ can be determined by eq. (21). From these values of T and θ, the predicted theoretical torque–twist curves shown in Figure 5 are found. A comparison of the actual and predicted torque–twist relations of Figure 5 shows good agreement.

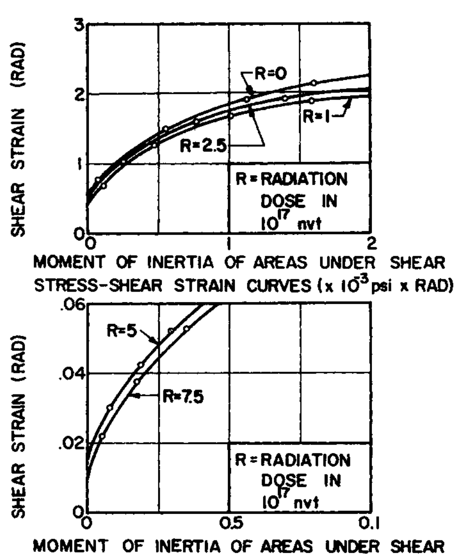


Fig. 19. Moments of inertia of areas under shear stress–strain for different radiation doses.

CONCLUSIONS

For the material and radiation doses considered, an increase in radiation produced the following effects:

1. Decreased elastic strength, fracture strength, ductility, resilience, and toughness, but increased stiffness, in *tension.*

2. Increased elastic strength, stiffness, and resilience, in *compression.*

3. Decreased modulus of rupture, ductility, and toughness, in *bending.*

4. Decreased elastic strength, modulus of rupture, ductility, resilience, and toughness, but increased stiffness, in *torsion.*

In general, the tests show a decrease in most short-time properties with an increase in radiation doses for tension, bending, and torsion, and an increase in short-time properties for compression.

This paper represents part of an M.S. thesis by P. B. Griesacker conducted under the supervision of J. Marin. The writers wish to thank Dr. Yoh-Han Pao of E. I. du Pont de Nemours & Company for supplying the test specimens used in this study.

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Synopsis

In this study, the influence of various nuclear radiation doses on the short-time tension, compression, bending and pure shear (torsion) stress–strain properties of polyethylene was investigated. In interpreting the test results, an attempt was made to determine the optimum radiation dose for each of the properties evaluated. In addition to the nominal tension and compression properties, the true stress–strain properties were determined. Nominal properties in bending and torsion were also found. For all four types of stress, the influence of radiation was found using four different doses of radiation. A second part of this paper deals with the theoretical prediction of bending-curvature and torque–twist relations from true stress–strain curves in tension and compression. A comparison of the predicted and actual relations for the various radiation doses considered shows good agreement between actual and predicted values.

Résumé

On a examiné dans ce travail l'influence de différentes doses de radiations nucléaires sur les propriétés du polyéthylène, telles que la tension instantanée, la compression, la courbure, les propriétés de tension-force de tension. En considérant les résultats de ces expériences, on a essayé de déterminer la dose optimale de radiation pour chacune des propriétés considérées. En plus des propriétés de tension et de compression, on a déterminé les propriétés exactes de la force de tension. On a également trouvé des propriétés nominales de courbure et de tension. On a trouvé l'influence de la radiation pour les quatres types de forces en utilisant quatre doses différentes de radiations. Une seconde partie de cette communication traite de la prévision théorique des relations pliage-courbure et couple de torsion–torsion à partir des courbes exactes de force de tension dans la tension et la compression. En comparant les relations prédites et réelles pour différentes doses de radiations on trouve une bonne concordance entre les valeurs réelles et prévues.

Zusammenfassung

In der vorliegenden Arbiet wurde der Einfluss verschiedener Kernstrahlungsdosen auf die Spannungs-Dehnungseigenschaften von Polyäthylen bei kurzzeitiger Spannungs-, Kompressions-, Beigungs- und reiner Scher(Torsions)beanspruchung untersucht. Anhand der Versuchserbegnisse wurde versucht die optimale Bestrahlungsdosis für jede dieser Eigenschaften zu bestimmen. Zusätzlich zu den nominellen Zug- und Druckeigenschaften wurden auch die wahren Spannungs-Dehnungseigenschaften bestimmt. Auch für Beigung und Torsion ergaben sich nominelle Eigenschaften. Bei allen vier Spannungstypen wurde der Einfluss der Bestrahlung durch Verwendung vier verschiedener Strahlungsdosen ermittelt. Im zwieten Teil der Arbeit werden aus wahren Spannungs-Dehnungskurven bei Zug- und Kompressionsbeanspruchung theoretische Angaben über die Biegungs-Krümmungs- und Verdrehungs-Verformungsbeziehungen gemacht. Ein Vergleich der theoretisch erhaltenen mit den tatsächlich bestehenden Beziehungen zeigt für die verwendeten Bestrahlungsdosen eine gute Übereinstimmung der experimentellen und theoretischen Werte.

Received September 11, 1961